A Fuzzy Multicriteria Mathematical Programming model for planning agricultural regions

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Jel classification: C610, Q150

Abstract

Modern agriculture is characterized by a series of conflicting optimization criteria that hinder the decision-making process in the planning of agricultural production. Such criteria are usually gross margin, the variance of gross margin that stands as a measurement of the total risk, labour, etc. At the same time, the decision-making process in the agricultural production planning is often conducted with data that accidentally occur in nature or that are fuzzy (not deterministic). Such data are the yields of various crops, the prices of products and raw materials, the available quantities of production factors, etc.

In this paper, a fuzzy multicriteria mathematical programming model is presented. This model is applied in a region of northern Greece with irrigated agriculture where the optimal production plan is achieved. Then, we compare the results of this model with those of a simple multicriteria mathematical programming model (MCDM model), as well as with those of a simple linear programming model (LP model).

Keywords: multicriteria models, optimal production plan, irrigated agriculture.

Résumé

L´agriculture moderne se caractérise par toute une série des critères d´optimisation en conflit qui entravent le processus décisionnel de planification de la production agricole. D´habitude, ces critères sont la marge brute, la variation de la marge brute qui est utilisée comme mesure du risque total, le travail, etc. En même temps, le processus décisionnel de planification de la production agricole se déroule souvent sur la base de données qui apparaissent accidentellement dans la nature ou qui sont floues (indéterminées). Ces données sont par exemple les rendements de différentes cultures, les prix des produits et des matières premières, les quantités disponibles des facteurs de production, etc.

Ce travail présente un modèle multicritère flou de programmation mathématique. Ce modèle est appliqué dans une région de la Grèce du Nord où l'agriculture est irriguée et le plan optimal de production est atteint. Ensuite, on compare les résultats de ce modèle aussi bien avec les résultats du modèle multicritère simple de programmation mathématique (modèle multicritère d'aide à la décision), qu'avec ceux du modèle élémentaire de programmation linéaire.

***Mots clés**: modèles multicritères, plan de production optimale, agriculture irriguée.*

1. Introduction

The simple models of linear programming for planning agricultural regions have been progressively substituted by more advanced and realistic mathematical programming models. At present, many models are used, among which multicriteria mathematical programming models are well known.

Multicriteria decision-making (MCDM) methodologies were introduced at the beginning of the 70s as important aid in the decision-making procedure. From then on, the number of contributions of MCDM in theory and models that are used as basis for decision-making with a more systematic and reasonable way has considerably increased. The number of methods developed proves the vigorousness of this discipline. There are lots of variations in MCDM methodologies, depending on their theoretical base. Zeleny (1982) argues that multicriteria analysis includes both multiple criteria and multiple goals. That is why there exist a couple of basic theoretical approaches, the multiple attribute utility theory (Manos *et al.*, 2009) and the multiple objective linear programming (Manos and Gavezos, 1995), both of which have been used as foundations for many theoretical variations.

When Zadeh (1965) and later Zimmerman (1978) introduced fuzzy sets in the MCDM discipline, they paved the way for a series of new methodologies that aimed at examining the problems unable to be solved with the already existing MCDM methodologies. The new methodologies were developed following the same directives. There are a lot of fuzzy MCDM studies (Lohani *et al.*, 2004; Panda *et al.*, 1996; Panigrahi and Majumda, 2000; Zimmerman, 1987; Zimmerman, 1990) that had a very important impact on the MCDM discipline; the most cited category is the so called fuzzy mathematical programming. Inuiguchi *et al.* (1990) made useful research with the recent developments in fuzzy mathematical programming.

In this paper, we present a fuzzy multicriteria mathematical programming model applied for the planning of an irrigated agricultural region in northern Greece. The optimal production plan satisfies three goals under a set of fuzzy constraints. The three goals unified to one single utility function are the gross margin maximization, the fertilizer use minimization and the minimization of labour used. The optimal plan achieves a higher gross margin (20.4%) and needs less fertilizer (-8.9%) and less labour (-9.2%).

The results achieved are compared with those of a simple multicriteria mathematical programming model (MCDM model), as well as with those of a simple linear program-

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ming model (LP). From this comparison, we conclude that fuzzy multicriteria mathematical programming is a more reliable method for planning agricultural regions with conflicting criteria based on fuzzy data.

2. Decision making in agricultural production planning using fuzzy data

For decision making in agricultural production, planning with fuzzy data and using mathematical programming, international literature suggests a number of methodologies based on the fuzzy sets theory.

The classical linear programming model has the following form:

$$z(x) = c_1x_1 + c_2x_2 + \ldots + c_nx_n \rightarrow Max$$

Subject to

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \leq b_i, \quad i = 1,2,\ldots,m,$$

$$x_j \geq 0, \quad j = 1,\ldots,n$$

However, in real problems the c_j, a_{ij} and b_i parameters do not always have deterministic values. Literature suggests a couple of different methodologies for dealing with this problem:

(i) Stochastic programming (SP) (Birge and Louveaux, 1997; Dupacova, 2002; Inuiguchi, 1992; Inuiguchi and Sakawa, 1995; Manos and Kitsopanidis, 1986; Rommelfanger, 2007) is used in cases where model parameters are random variables. A SP model has the following form:

$$z(x,\omega) = c_1(\omega)\cdot x_1 + c_2(\omega)\cdot x_2 + \ldots + c_n(\omega)\cdot x_n \rightarrow Max$$

subject to

$$a_{i1}(\omega)\cdot x_1 + a_{i2}(\omega)\cdot x_2 + \ldots + a_{in}(\omega)x_n \leq b_i(\omega), \quad i = 1,2,\ldots,m,$$

$$x_j \geq 0, \quad j = 1,\ldots,n,$$

where $c_j(\omega)$, $a_{ij}(\omega)$ and $b_i(\omega)$ are random variables with a concrete interval of probability.

In most of the well known models of stochastic programming in agricultural production planning, the goal is:

a) the optimization of the objective function that usually represents the mean value of the total gross margin in a specific time horizon $Max_x\ E(z(x,\omega))$, or

b) the minimization of the variance of the total gross margin $Min_x\ Var(z(x,\omega))$, or

c) the minimum risk problems $Max_x P(\omega|z(x,\omega)\geq\gamma$, where γ is a fixed level.

These models are difficult to be solved and are used only for specific probability distributions that lead to an equivalent model which can be solved. Especially models of forms b and c are non linear and only some of them are able to be solved, e.g. quadratic programming models (Manos and Kitsopanidis, 1986).

(ii) Fuzzy programming (FP) (Delgado *et al.,* 1989; Inuiguchi, 1992; Rommelfanger, 1995; Rommelfanger, 2004; Rommelfanger and Slowinski, 1998; Yazenin, 1992; Zimmerman, 1992; Zangiabadi and Maleki, 2007; Rommelfanger, 2007) is used when the uncertain values are fuzzy sets. A FP model has the following form:

$$\tilde{Z}(x) = \tilde{C}_1x_1 + \tilde{C}_2x_2 + \ldots + \tilde{C}_nx_n \rightarrow Max$$

subject to

$$\tilde{A}_{i1}x_1 + \tilde{A}_{i2}x_2 + \ldots + \tilde{A}_{in}x_n \leq \tilde{B}_i, i = 1,2,\ldots,m,$$

$$x_j \geq 0, \quad j = 1,\ldots,n,$$

where $\tilde{C}_j$, $\tilde{A}_{ij}$, $\tilde{B}_i$ are fuzzy sets R.

For cases where the fuzzy values are fuzzy numbers or fuzzy spaces of the same L-R type for every constraint, literature suggests several methodologies for solving the FP models.

Many authors in international literature proceed further and compare the results achieved by the two methodologies of stochastic programming and fuzzy programming (Inuiguchi, 1992; Rommelfanger, 1990; Yazenin, 1987).

3. Fuzzy multicriteria linear programming methodology (FMCDM)

The decision making procedure in the planning of agricultural production has a number of conflicting criteria like gross margin, the variance of gross margin, labour, etc. It also uses data randomly arising in nature or fuzzy, like the yields of crops, the prices of raw materials and of agricultural products, the available quantities of production factors, etc.

In order to handle in a more efficient way the conflicting criteria in the decision making process, there are a number of available techniques (Kuol and Liu, 2003; Majumdar, 2002). Optimization with the use of fuzzy linear programming techniques can deal with the problem of uncertainty due to fuzzy data (Toyonaga *et al*., 2005; Yin *et al.,* 1999). The approach of fuzzy sets theory provides a hopeful alternative solution to the existing decision making methodologies and allows for the integration of expert's experience.

In this paper, we attempt to achieve the optimal use of agricultural land and other resources in an irrigated agricultural area with a number of different cultivated crops, using fuzzy multicriteria mathematical programming (Sahoo *et al.,* 2006) and taking into account three conflicting goals: maximization of gross margin, minimization of fertilizers use and minimization of labour.

To solve this problem, we initially define the three independent goals and the multiple goals problem and then the model is adjusted to the real conditions with the integration of the interdependencies between the goals. Interdependencies among the goals come into focus when the computerised value of the objective function is not equal with the existing value. At first, the goals are classified based on their importance to the producers. Then, we solve the first goal: Z_1 = max $\{f_1(X)$: based on the set of constraints$\}$.

Then, for every $i > 1$, $Z_i = \max \{f_i(X): f_k(X) = Z_k(X)$ for $k = 1,.., i - l\}$, where Z_i and f_i are the objective function and the constraints respectively. This methodology would be especially useful if there were more than one X for $Z_1(X)$ and if exists a predefined weighted scale for the goals.

More precisely in our study we adopted the following solving procedure applied by Sahoo *et al.* (2006):

α) The goals are ordered based on their weight.

The FMCDM takes into account the three fuzzy objective functions, in order to examine the problem of optimum crops plan and resources allocation in the study area. The conflicting criteria and fuzzy decision-making procedure are as follows:

$$\textit{Maximize} \quad Z_{GM} = f_1(x_1, x_2, \ldots, x_n)$$

$$\textit{Minimize} \quad Z_{FER} = f_2(x_1, x_2, \ldots, x_n)$$

$$\textit{Minimize} \quad Z_{TL} = f_3(x_1, x_2, \ldots, x_n)$$

where Z_{GM}, Z_{FER} and Z_{TL} are the fuzzy goals for the maximization of gross margin, the minimization of fertilizers use and the minimization of required labour respectively. Variables X_1, X_2, ...X_n are assigned to the crops cultivated in the area and correspond to those of table 1.

The fuzzy goal $f(X_1, X_2, ... X_n) \geq k$, which is called a fuzzy set **f** and is defined beyond the set of feasible solutions, is presented by functions μ_1, μ_2 and μ_3, which are linearly defined with the following forms:

$$\mu_1 = \int_0^1 \frac{f_1(x_1, x_2, \ldots, x_n) - P_{TL}}{Z_{GM} - P_{TL}}, \qquad \begin{array}{c} f_1(x_1, x_2, \ldots, x_n) > Z_{GM} \\ P_{TL} \leq f_1(X_1, X_2, \ldots, X_n) \leq Z_{GM} \\ f_1(x_1, x_2, \ldots, x_n) < P_{TL} \end{array}$$

$$\mu_2 = \int_0^1 \frac{f_2(x_1, x_2, \ldots, x_n) - B_{TL}}{Z_{FER} - B_{TL}}, \qquad \begin{array}{c} f_2(x_1, x_2, \ldots, x_n) > Z_{FER} \\ B_{TL} \leq f_2(x_1, x_2, \ldots, x_n) \leq Z_{FER} \\ f_2(x_1, x_2, \ldots, x_n) < B_{TL} \end{array}$$

$$\mu_3 = \int_0^1 \frac{f_3(x_1, x_2, \ldots, x_n) - L_{GM}}{Z_{TL} - L_{GM}}, \qquad \begin{array}{c} f_3(x_1, x_2, \ldots, x_n) > Z_{TL} \\ L_{GM} \leq f_2(x_1, x_2, \ldots, x_n) \leq Z_{TL} \\ f_3(x_1, x_2, \ldots, x_n) < L_{GM} \end{array}$$

Where P_{TL} = the value of gross margin corresponding to the optimum solution for minimum required labour, B_{TL} = the value of used fertilizers that corresponds to the optimum solution for minimum total required labour and L_{GM} = the value of required labour that corresponds to the optimum solution for gross margin maximization.

The functions μ_1, μ_2 and μ_3 of the fuzzy sets describe the objective function that is in linear form and with values that vary from [0, 1] till the maximum possible value of Z_{GM}, Z_{FER} and Z_{TL}. The marginal conditions of the functions represent two extreme scenarios, e.g. μ_i $(i = 1, 2, 3) = 1$ indicates the total satisfaction of all the three goals and μ_i $(i = 1, 2, 3) = 0$ indicates the total conflict between the goals, one of which at least has zero satisfaction.

β) In continuation, the fuzzy decision Z is defined by the function:

$$Z = Z_{GM} \cap Z_{FER} \cap Z_{TL} \cap \mu_1 \cap \mu_2 \cap \mu_3$$

Thus, the optimum model is derived as follows:

Max Z

Subject to:

$Z \leq \mu_i$ $(i = 1, 2, 3)$

together with a set of other constraints defined in the following sections.

4. Model implementation

We implemented the fuzzy multicriteria mathematical programming model in the Sarigkiol basin in the prefecture of Kozani that covers an area of 407 km^2. The study area is part of the Sarigkiol basin between two municipalities of the prefecture of Kozani and consists of irrigated land of about 20,000 hectares. All the technical and economic data that were used in the model were gathered from the study area. The area is characterized by a Mediterranean climate with an average annual temperature of 12.96 °C and annual rainfall of 643 mm.

Table 1 – *Existent production plan.*

No	Crops	(ha)	(%)
1	Soft wheat	1,324	6.5%
2	Hard wheat	12,411	61.1%
3	Barley	1,269	6.3%
4	Corn	1,809	8.9%
5	Sugar beets	1,997	9.8%
6	Oat	76	0.4%
7	Potatoes	252	1.2%
8	Set aside	1,164	5.7%
	Total	20,302	100.0

The main crops cultivated in the study area are durum and soft wheat, corn, sugar beets, potatoes, oat and barley. The existent crop production plan in the study area is presented in table 1.

The optimum production plan must satisfy on the one hand the producers interest in the maximization of gross margin and minimization of labour and on the other hand the local community's interest in the minimization of the environmental impact by the use of fertilizers. Initially, those goals were achieved one by one individually as optimal solutions of three different simple linear programming models.

Goals

In particular, the three goals that were optimized were:

a) Maximization of gross margin

All farmers wish to maximize the gross margin of their agricultural production. The objective function included in the model is defined as follows:

$$\max \; GM=\sum GM_i \times X_i$$

b) Minimization of fertilizers

The minimization of fertilizer use is a societal goal. For this reason it is not taken into account during the decision making process by the farmers and is a conflicting goal. The minimization of fertilizers use is the main way to determine the surplus of nitrogen released that is potentially dangerous for the environment. It is also the main figure for the impact of agriculture in the environment, as groundwater quality is affected by fertilizer use.

$$\min \; FER=\sum FER_i \times X_i$$

c) Minimization of labour

The total labour demand (TL) is computed as the total labour required for all agricultural activities (X_i), therefore the objective function should be:

$$\min \; TL=\sum TL_i \times X_i$$

Constraints

In our model, we used various constraints concerning

- *Total agricultural area*
- *Common Agricultural Policy*
- *Market and other constraints*
- *Set aside and agronomical constraints*

Attributes

We also included some attributes not taken into account by the farmers during the decision making process; these attributes present a major interest for the analysts. The attributes included in the model are:

a) Water consumption: the consumption of water measured in m^3/ha is the attribute policy makers wish to control, in order to adhere to the changes in water management policies.

b) Environmental impacts: the main environmental impact of irrigated agriculture is water consumption as elaborated earlier, but also fertilizers and other chemicals application that constitutes the main source of underground water pollution. We use fertilizer consumption as an indicator for the environmental impact of irrigated agriculture, measured in nitrogen kilograms released per hectare (N/ha).

5. Results

It is obvious that we try to achieve a solution to the objective function in a fuzzy environment using 3 goals that are led to a unified solution through a set of fuzzy constraints. The objective functions of the linear model are gross margin maximization, fertilizer use minimization and the labour minimization.

The aim is to use the available agricultural land and other available resources in such a way to achieve the maximum gross margin with the minimum use of fertilizers and labour. The above goals are depended on water availability, capital, fertilizers and the existence of proper dry and irrigated crops in the study area. All three simple linear programming models use available agricultural land in a manner suitable as to achieve the optimum production plan based on the above goal. In order to achieve the optimum fuzzy solution the results of the simple linear programming model are used for the expression of the fuzzy constraints.

The results are in table 2 which presents the different production plans based on the optimal solution obtained by simple linear programming (LP), multicriteria linear programming (MCDM) and fuzzy multicriteria linear programming (FMCDM).

Table 2 – *Study area production plan using LP, MCDM and FMCDM.*

	Simple Linear Model (LP)			MCDM	FMCDM
	GM Max	FER Min	LAB Min		
Soft wheat	1.7	1.7	13.2	0.0	4.0
Barley	9.5	9.5	0.0	4.9	4.9
Durum wheat	61.1	61.1	61.1	61.1	61.1
Corn	13.0	16.9	16.6	17.8	19.0
Sugar beets	0	0	0	4.6	0
Oat	2	2	0	2.0	2.0
Potatoes	4.2	0	0	1.2	2.0
Set aside	8.5	8.8	9.1	8.4	7.0

The percentage differences among the 2 multicriteria programming models from the existent plan are presented in table 3. It emerges that using both the MCDM model and the fuzzy FMCDM model, gross margin increases, satisfying the first criterion concerning the farmers, which is the maximization of gross margin. The MCDM model results in a 9.2% increase, while the FMCDM model results in a 20.4% increase. At the same time, the societal goal, which is the decrease in environmental impacts through the reduction of fertilizers use, is also satisfied. In this case, the deduction is -4.8% for the MCDM model and -8.9% for the FMCDM model, with respect to the existent plan. The third goal, as from table 3, can only be satisfied by the FMCDM model, as the MCDM model produces no difference with the labour demand of the existent plan, while the FMCDM model achieves a -9.2% decrease.

As far as the production plan is concerned, notable are the variations regarding sugar beets, durum wheat, potatoes and not cultivated agricultural land left for set aside. We observe that the FMCDM model suggests the total abandonment of the sugar beets cultivation, something in accordance with current conditions, as it is known that this cultivation is no longer subsidized by the revised CAP. At the same time, the FMCDM model suggests to increase the cultivation of potatoes in the area by 66.6%, a development consistent with the local area conditions, where certain municipalities are well known for the good quality of the locally produced potatoes.

Table 3 – *Gross margin, fertilizer use, labour demand and percentage variations from the existent plan using MCDM and FMCDM.*

	Existent plan	MCDM model		FMCDM model	
		Value	% variation	Value	% variation
GM	49,330	53,903	9.2	59,414.2	20.4
FER	58,678	55,856	-4.8	53,414.3	-8.9
TL	8,962	8,962	0.0	8,132.6	-9.2
Soft wheat	6.5	0.0	-100.0	4.0	-38.4
Barley	6.3	4.9	-22.2	4.9	-22.2
Durum wheat	61.1	61.1	0.0	61.1	0.0
Corn	8.9	17.8	100.0	19.0	113.4
Sugar beets	9.8	4.6	-53.0	0	-100.0
Oat	0.4	2.0	400.0	2.0	400.0
Potatoes	1.2	1.2	0.0	2.0	66.6
Set aside	5.7	8.4	47.3	7.0	22.8
Total	100.0	100.0		100.0	

6. Conclusions

We developed a fuzzy multicriteria mathematical programming model (FMCDM model) with 3 conflicting goals in a unified objective function. Afterwards, this model was applied in the agricultural production planning of an irrigated region in northern Greece. The results were compared with those of the simple multicriteria mathematical programming model (MCDM model) and with those of the simple linear programming model (LP). We observed that the FMCDM model satisfies all 3 goals we set, differently from the MCDM model that satisfies only 2 out of the 3 goals and the LP model that is able to satisfy only one independent goal each time. As for the existent production plan, we noted that the FMCDM model proposes changes that are in accordance with the recently revised CAP as well as the with special conditions of the study area.

From the results, we conclude that fuzzy multicriteria mathematical programming is a more reliable method for planning agricultural production. Moreover, it can investigate more successfully matters involving conflicting criteria in agricultural production planning, that is mostly based on fuzzy data. The proposed model could easily be extended and include more variables, constraints and attributes, such as the quality of water or the possible changes in its pricing.

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